**Assignment 5: Quicksort Algorithm: Implementation, Analysis, and Randomisation**

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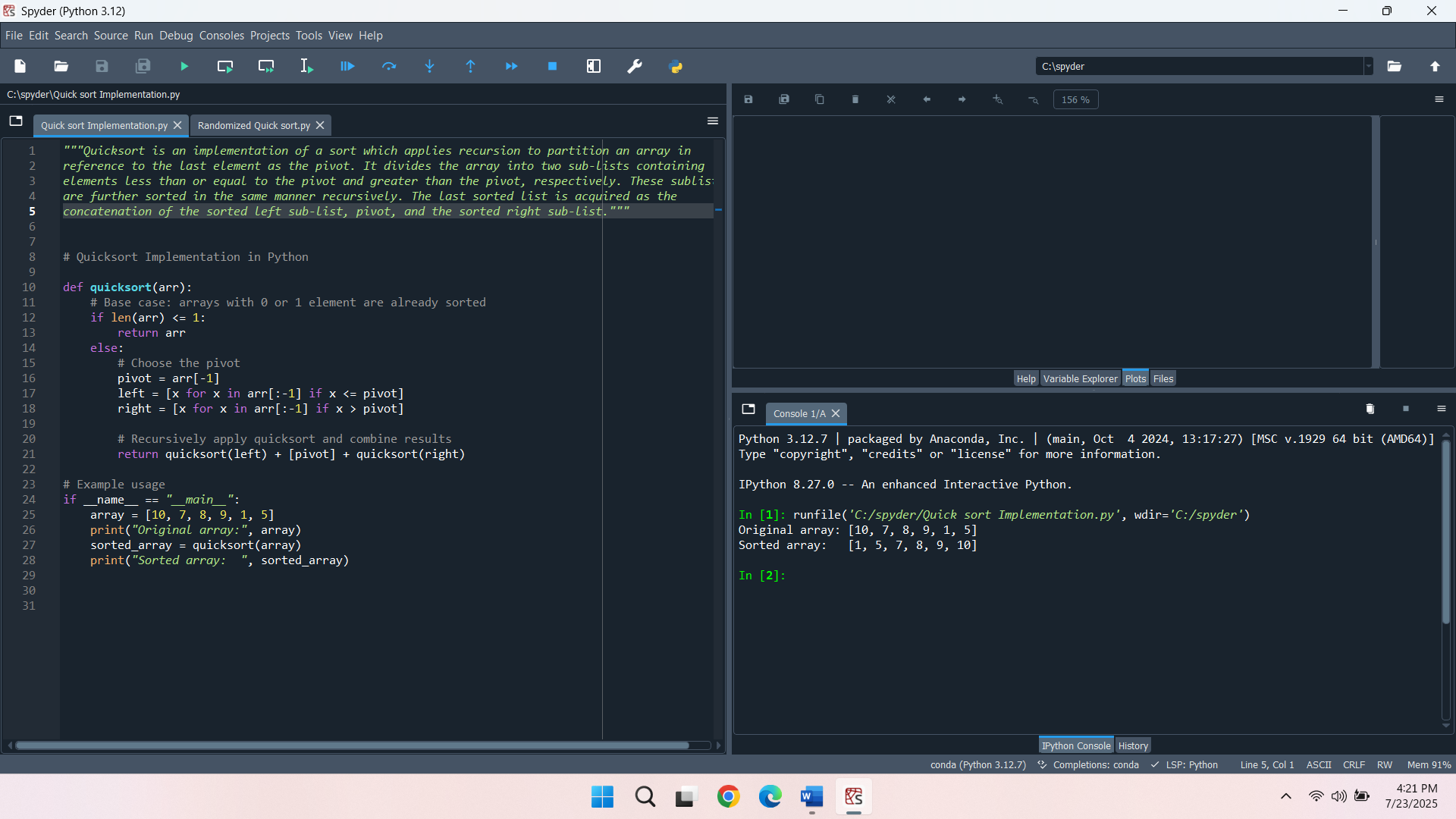
**Introduction**

Quicksort is an effective sorting algorithm with broad application that implements the divide-and-conquer approach. Known for its average-case time complexity of O(nlogn). Quicksort is a desirable algorithm in numerous real-world settings, such as system-level libraries to the processing of large amounts of data. This piece of work will examine both the deterministic and the randomised versions of Quicksort, both in terms of implementation, performance analysis, and their comparison under different test scenarios. Focusing on theoretical research and practical experiments, the assignment identifies the advantages and drawbacks of both directions and underlines the necessity to focus on the selection of pivots that can guarantee the best performance.

**Quicksort Implementation and Analysis**

**1. Implementation**

Quicksort is an implementation of a sort which applies recursion to partition an array in reference to the last element as the pivot. It divides the array into two sub-lists containing elements less than or equal to the pivot and greater than the pivot, respectively. These sub-lists are further sorted in the same manner recursively (Aftab, 2021). The last sorted list is acquired as the concatenation of the sorted left sub-list, pivot, and the sorted right sub-list.



**2. Performance Analysis**

**Time Complexity of Quicksort**

* **Best Case O(nlogn):** An ideal situation occurs when at each level of call, the pivot partitions the array into two halves. This results in a balanced recursion tree with logn depth, and each level processes all n elements during partitioning, leading to O(nlogn) overall.
* **Average Case - O(nlogn):** The pivot separates the array into reasonably balanced sections on average. The splits are not always best, but good enough to maintain the depth around logn. Since each level still scans the full array for partitioning, the total time complexity remains O(nlogn).
* **Worst Case - O(n2):** The worst time complexity is when the pivot is the minimum or maximum in every iteration, leading one subarray to have n-1 elements and the other having 0 elements. This leads to a recursion depth of n and a total of n+(n−1)+(n−2)+...+1=O(n2) comparisons.

**The Average-Case Time Complexity is O(nlogn) and the Worst Case is O(n2):**

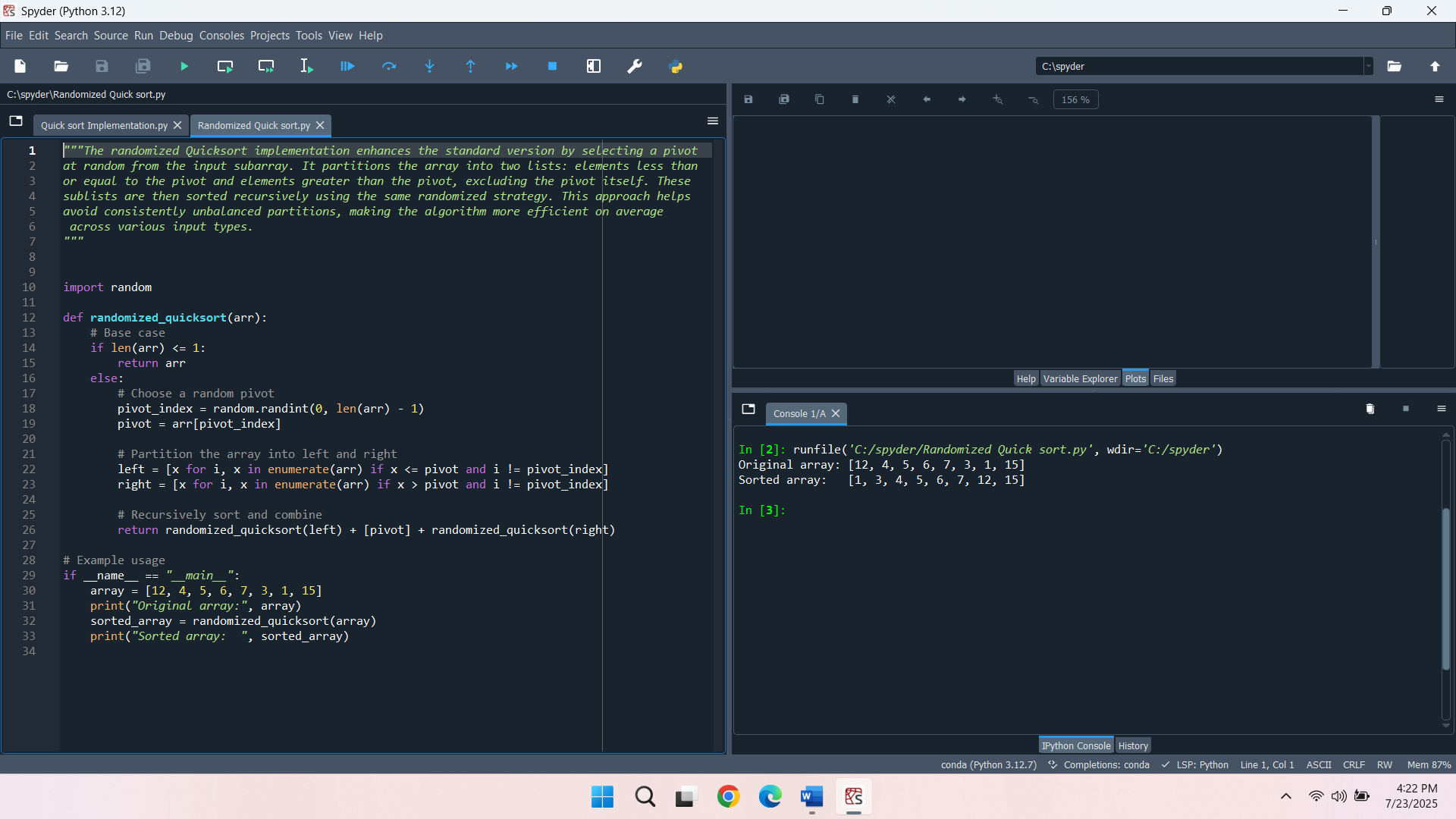
The average-case complexity is O(nlogn) because, with random or unsorted data, the pivot tends to split the array into reasonably balanced subarrays over time. This balanced recursion tree is logn deep, and n partitions take place in each step (Kalmodiya & Dixit, 2024). In contrast, the worst-case O(n2) arises when the pivot causes highly unbalanced splits, leading to a linear-depth tree with n levels, where each level processes nearly all remaining elements.

**Space Complexity and Additional Overheads:**

* **Space Complexity:** In the best and average cases, the recursive depth is logn, so the space complexity is O(logn) due to the function call stack. In the worst case, the recursive depth becomes n, leading to a space complexity of O(n).
* **Additional Overheads:** Minimal overhead because Quicksort is an in-place sorting algorithm, it does not have to use additional arrays such as Merge Sort. Nevertheless, it suffers overhead due to recursion, and it may suffer stack overflows and grow slowly when using poor pivot choices. This risk can be mitigated by selecting pivots at random or using a median of three strategies.

**3. Randomised Quicksort**

The randomised Quicksort version improves the simple one since it randomly chooses a pivot within the input subarray. It divides the array into two lists: elements less than or equal to the pivot and elements greater than the pivot, not the pivot itself. These sub-lists are optimised again with the same randomised technique used in sorting recursively. The method allows for avoiding repeatedly lopsided partitions, so the algorithm is on average more efficient over a wide range of inputs (Wassenberg, 2024).



**Analysis**

Randomisation significantly reduces the chance of Quicksort hitting its worst-case time complexity of O(n2). In the deterministic model, poor pivot selection, such as smallest or largest element in sorted or almost sorted arrays, however, always causes unbalanced partitions. Nonetheless, when a pivot is chosen randomly, the chances of disproportionately unbalanced splits between recursive calls are negligibly small.

As a result, randomised Quicksort has an expected time complexity of O(nlogn) even for adversarial inputs. It is strong and effective to use in practice, particularly where the distribution of inputs is unseen or perhaps designed in such a manner as is otherwise likely to make the performance worse. The randomness averages out average-case performance in more consistent data types.

**4. Empirical Analysis**

**Empirical Comparison of Running Time**

To compare deterministic and randomised variants of the Quicksort algorithm, they were applied to arrays of various sizes and distributions. The input types in this analysis were:

* Random arrays
* Sorted arrays
* Reverse-sorted arrays

To scale the performance, arrays having different sizes were applied in each type of input. Standard timing functions were used to time the execution of each of the algorithms several times to obtain consistency.

The deterministic Quicksort, which uses a fixed pivot, performed efficiently on random inputs, typically showing time complexity close to O(nlogn). Nevertheless, the algorithm indicated severe performance deterioration when the input was sorted or reverse-sorted: in this case, the partitions were always unbalanced. These scenarios led to near O(n2) behaviour, especially as input size increased.

Conversely, the randomised Quicksort was always stable with all input data. The algorithm worked by making a random decision on a pivot to prevent unnecessary recurrent bad partition. Even on sorted or reverse-sorted arrays, randomised Quicksort maintained an average time complexity close to O(nlogn), with much lower execution times compared to the deterministic version under those conditions.

**Discussion of Observed Results and Relation to Theoretical Analysis**

The presented outcomes easily reinforce the theoretical study of the Quicksort performance. As predicted, deterministic Quicksort is vulnerable to input patterns where the fixed pivot leads to unbalanced splits, resulting in a worst-case time complexity of O(n2). This could be seen in the experiment done on the sorted and reverse-sorted arrays, where input size grew in leaps and bounds with the size of input.

Randomised Quicksort, however, has been shown to have robust and efficient behaviour against all input types. The randomness over learning pivots minimises the likelihood of always making bad splits, thus minimising the likelihood of worst-case behaviour. This aligns with the theoretical expectation that randomised Quicksort has an expected time complexity of O(nlogn) regardless of the input's initial order.

To sum up, deterministic Quicksort is only likely to work well with random data, but its randomised version is more trustworthy and can grow with scale, being therefore more applicable to those cases when it is impossible to ensure the input characteristics.

**Conclusion**

Quicksort, both in deterministic and randomised forms, shows how extraordinarily enormous the role of pivot selection can be to determine the efficiency of an algorithm. The deterministic version is very good on randomly ordered inputs, but very bad on sorted and reverse-sorted on unbalanced partitioning. Randomised Quicksort avoids this disadvantage by choosing pivots randomly and, as a result, demonstrates near-optimal performance on all kinds of input. The theoretical results are justified by empirical analysis, and randomisation indeed makes the algorithm more robust and safer against the worst-case behaviour. All in all, there should be an analysis of these varieties of Quicksort to understand and apply them all to give the developers and data scientists the equipment to construct faster, more scalable systems.

**References**

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